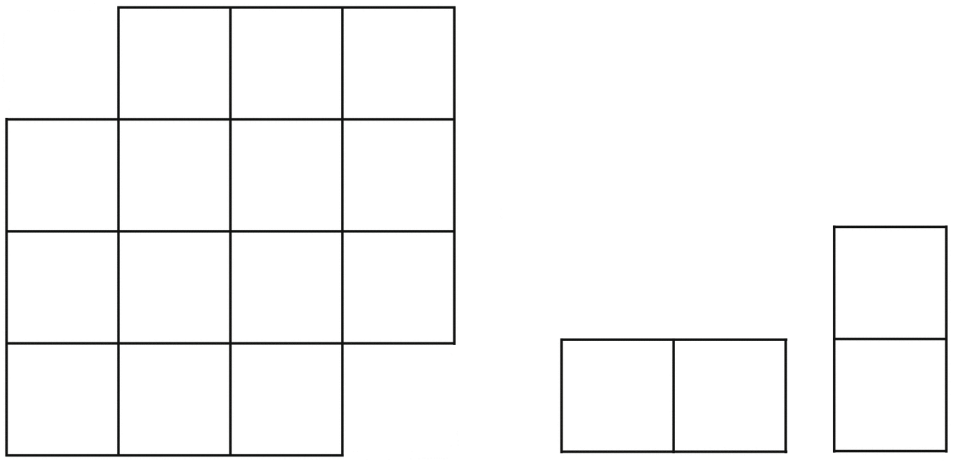
Maths Circle Explorations: Session 6

TIFR, Mumbai

7th January 2022

Problem 3

1. Suppose you are given a 4×4 square grid with squares at two diagonally opposite corners removed. In how many ways can you tile this truncated grid with combinations of 2×1 and 1×2 dominoes?



(a) The Truncated 4×4 grid. (b) The dominoes allowed.

Figure 1

2. A knight is in a battle with a mythical dragon with exactly 100 heads. With one swoop of the sword, the knight can cut off either 15, 17, 5 or 20 heads. The dragon dies if all of its heads are cut. But otherwise, in each of the four respective cases, the dragon grows back 24, 2, 14 or 17 heads.

   Can the knight ever defeat the dragon?

3. In a dinner party, $2n$ guests are supposed to sit at a round table, where each seat is next to two others. Each guest has at most $n - 1$ enemies among the invited. Is there a seating arrangement where none of the neighbours of each guest is an enemy?