

Maths Circle Explorations: Session 8

IISER Mohali

4th February 2022

Problem 3

1. Amit and Smita play a series of games where in each game they can either win or lose one rupee. In each round, Amit earns Rs. 1 with probability p and Smita loses Rs. 1. Conversely, Amit loses Rs. 1 with probability $1-p$ and Smita gains Rs. 1. Suppose Amit starts with Rs. n . Find the probability that Amit loses all his money.
2. A person X walks on Z starting at 0. At every time-step, she moves one step towards the right with probability p and one step towards the left with probability $1-p$.
 - (a) What is the probability that she ever returns to 0?
 - (b) Take $p = 1/2$. Suppose she is at n and has not yet returned to 0. What is the probability that she returns to 0 within $2n$ steps?