

Maths Circle Explorations: Session 3

November 26, 2021

3. (a) Convince yourself that the bagel shaped object in Figure 2 below can be constructed by taking the sheet of elastic material shown on the right, and gluing together the edges with identical labels (in such a way that the directions of the arrows on the edges match up). Compare this with Activity 2 of Session 1, and the doughnut shaped chessboard of Activity 3 in Session 2.

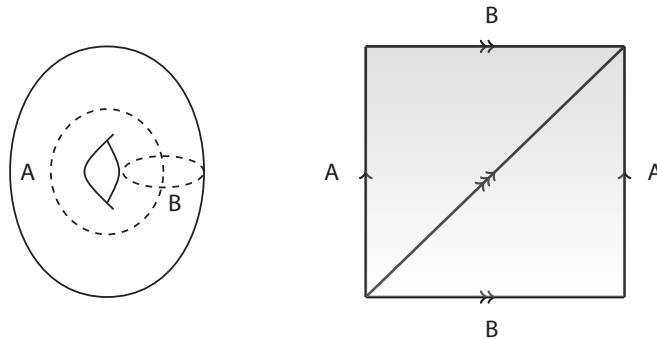


Figure 2: Constructing a torus from a sheet of elastic material

- (b) Figure 3 shows “pretzel-like” surfaces with 2 “holes” and 3 holes. Can you think of a way to construct these surfaces that is analogous to the construction of the surface of a bagel in part (a)? That is, can you find a way to cut out a shape from a thin sheet of elastic material, and glue together various boundary edges to get the shapes shown in Figure 3? Does your construction work if the pretzel has 4 or more holes?

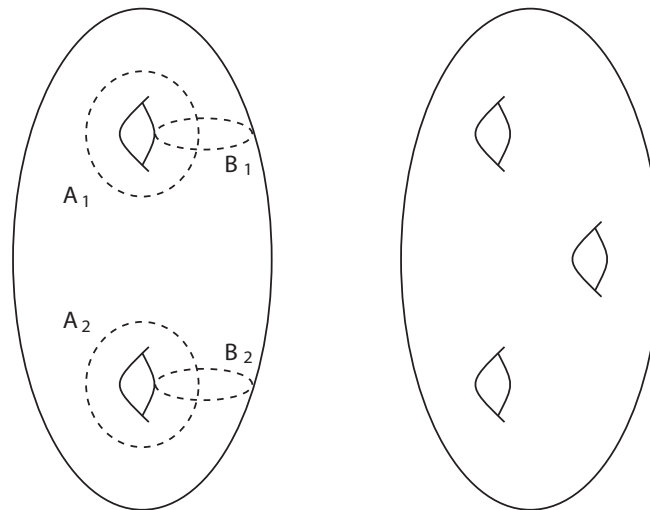


Figure 3: The surface of a bagel with extra “handles” attached.

- (c) Find a way to cut up the surfaces in part (b) into triangles, as was done for the bagel in part (a). Count the number of vertices, edges and triangles that you get. How are they related to the number of holes in the pretzel? How does your answer change if you chop up the surface into triangles in a different way?
- (d) Suppose that the material that the pretzels are made of behaves exactly like the material described in Activity 2 above. Is it possible to deform the pretzel with 2 holes into a bagel? Can one deform it into a pretzel with 3 holes? Give a convincing argument to justify your answer.