

Math Circle Explorations: Session 3

Room A

Problem 1. Observe that for some small integers, if we attach some extra digits on the right hand side, we can “complete” them to a power of 2. For example, for the integer 1, we write 6 on the right to get $16 = 2^4$. Similarly, we write 56 to the right of 2 to get $256 = 2^8$. Some more examples:

$$\begin{array}{l} 3 \quad 32 = 2^5 \\ 4 \quad 4096 = 2^{12} \\ 5 \quad 512 = 2^9 \\ 6 \quad 64 = 2^6 \dots \end{array}$$

Seems easy enough... right? Next one is a bit harder.

$$7 \quad 70,368,744,177,664 = 2^{46}$$

That took some work!

$$\begin{array}{l} 8 \quad 8192 = 2^{13} \\ 9 \quad 9007199254740992 = 2^{53} \end{array}$$

Almost gave up there!

$$\begin{array}{l} 10 \quad 1024 = 2^{10} \\ 11 \quad 1125899906842624 = 2^{50} \\ 12 \quad 1208925819614629174706176 = 2^{80} \end{array}$$

Phew!

$$13 \quad 131072 = 2^{17}$$

... and so on.

Do you think that any integer can be “completed” to a power of two by writing some more digits to the right? Or is it impossible to do this for some integers?