Math Circle Explorations: Session 3

Problem 2. Let x be a real number greater than 1. We consider the sequence $\lfloor x \rfloor, \lfloor 2x \rfloor, \lfloor 3x \rfloor \ldots$ and so on. (For any real number y, $\lfloor y \rfloor$ denotes the greatest integer less than or equal to y. Thus, $\lfloor 2.73 \rfloor = 2$, $\lfloor 3 \rfloor = 3$.) Let us denote this sequence by S_x.

How do these sequences behave? Given real numbers x and y, both greater than 1, is there any way to predict if S_x and S_y will have some common terms? Is there any way to predict if they will have no common terms?