Math Circle Explorations: Session 3

Problem 1. Do the following:

a) Let θ be a rational number between $(0, 360)$. Consider a circle of unit radius centred at origin in the plane. Start with the point $(1, 0)$ and rotate it in the anticlockwise direction by θ degrees. Let us again rotate the new position by θ degrees and repeat this process. Will you ever hit the initial positon $(1, 0)$ if at each step you rotate by θ degrees? Will you hit the initial position multiple number of times?

b) Now pick θ to be any irrational number between $(0, 360)$. Let us proceed in a similar manner as in the previous part. Will you ever hit the initial position $(1, 0)$ in this case?

c) Let S be the collection of all points on rotation by θ degrees starting from $(1, 0)$ as above. Pick any interval I having more than one point on the unit circle centred at the origin. When will be the intersection non-empty for any interval I with S on the given circle? Does the choice of θ matter here?