

Maths Circle India

TIFR-STCS Maths Circle Team

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4 Moving ahead

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The kingdom of Phoolpatti is now no more at war, and the citizens are now organising their annual games festival. The queen, now freed from worrying about wars and bridges, is also back at her hobby of designing mathematical games. Her latest game takes the following form. First, she creates a map which must satisfy certain conditions. Here are the conditions.

1. A map consisting of just one point is a valid map. In this case, this single point is called the root of the map.
2. If M_1 and M_2 are two valid maps with roots at points p_1 and p_2 respectively, then the map M comprising, in addition to the old maps M_1 and M_2 , a new point r along with arrows $r \rightarrow p_1$ and $r \rightarrow p_2$ is also a valid map. The new point r is then declared to be the root of M .

Can you create some such maps? How many maps are there with just one point? Just two points? Just three points?... Just eight points?

Once she has such a map M , the game is played between two players: the painter and the planner. First, note that for any point p in a valid map M , there are either zero or two arrows going out of p . The painter's job is to colour one arrow blue and the other red, at each point p in the map which has two arrows going out. Note that the painter can choose at each point which of the two arrows he would colour red and which he would colour blue.

Once the painter is done painting, the planner's job is to look at the painting and ask the painter to "travel" from the root of the map M to one of the points in the map that has no arrows going out of it. The planner can choose any such point, and the painter can only travel in the direction of the arrows. The planner's score is the number of red arrows that the painter has to follow.

The painter's goal in the game is to minimize the planner's score, while the planner's goal (of course) is to maximize her score.

What is the best the painter can do in the example maps you constructed? Is there a simple method for finding out this "best possible score" given the description of the map?