

Maths Circle India

TIFR-STCS Maths Circle Team

Session 3: May 20, 2022

2 Target chasing

During his summer vacations, Lava has developed a new solitaire game (i.e., a game that one can play alone, without any opponents) that goes as follows. He draws two long perpendicular line segments on a piece of paper, one horizontal, and the other vertical, which intersect at a point O . Let the right-most point on the horizontal line segment be A . He then draws an arrow, called target of length 5cm with its tail at O , and calls its head T . Let the angle AOT be called φ .

He then draws another, called chaser of length 5cm with its tail at O , and calls its head C . Let the angle AOC be called θ .

In each move of the game, only the chaser is moved: the target remains fixed in place. Each such move consists of two moves:

1. First, reflect the chaser in the horizontal line.
2. Then, reflect the chaser (in its new position) in the target line.

So, how does the game progress when the chaser is initially 70° "below" the horizontal line (so $\theta = -70^\circ$), and the target is 10° above the horizontal line (so $\varphi = 10^\circ$)? How about when the initial positions are -70° and 5° respectively? Can you give a general "rule" for figuring out what will happen?