

Maths Circle India

TIFR-STCS Maths Circle Team

Session 5: June 24, 2022

2 Another party at Zoya's house

Credits: Shklyarky, Chentsov, and Yaglom, Selected problems and theorems in elementary mathematics (Mir Publishers, Moscow 1979)

After the success of Zoya's first party, Zoya has gotten fond of the idea of throwing parties. This time, she wants to have her party in the little garden at the back of her house and invite 20 of her classmates. Her garden has a round stone table with four stone seats set around it. She wants to ensure that she can find four of her classmates to sit around this table such that each person sitting on one of the stone seats will be friends with their neighbour on both sides of them. Remembering the difficult time she had figuring out how many people to invite to her last party ((in which she wanted to make sure that the three-seater table in her house was occupied by three classmates who were all friends with each other or by three classmates none of whom were friends with the other two), she starts wondering if 20 is a large enough number to guarantee that she will be able to do this.

But then she remembers that each of these 20 classmates of hers is friends with at least 10 others in this group of 20. And she stops worrying. Why?