DTP-Math-Circle: Session 3—Conditional probability

Sept 30 2022

4 Monty Hall problem

This is based on the famous problem from the Monty Hall game show. Monty shows you three doors. Behind one of these doors is a car, which you will win if you open that door. Behind the other two doors are goats, which you can pet for a while, but not take home.

(a) You choose a door first. Monty then says "I shall not open that door. But to help you, I shall now open another door behind which there there is a goat," and does so. Now there are two closed doors, behind one is a car and behind the other is a goat.

Monty now gives you the option of switching the door you selected earlier. Assuming that you want to maximise the chances of winning the car, should you switch or stay with your earlier choice ?

(b) Suppose Monty says "I shall not open the door you choose, but shall open a random other door". He opens it and there is a goat behind it. Now should you switch?

(c) Suppose Monty says "I shall now open a random door from these three". He opens one (which is not the one you had chosen) and there is a goat behind it. Now should you switch?

(d) In the 100th Monty Hall show, there were 100 doors, with a goat behind 99 doors and a car behind one door. This time Monty tells you that he will not open the door you have chosen, but will open 98 others which have a goat behind each of them, and goes on to do so. Now again there are two closed doors, one originally chosen by you. What is the probability that the car is behind your door? Behind the other closed door?