DTP-Math-Circle: Session 3—Conditional probability

Sept 30 2022

6 Duels and Three-way paintball

Two paintball shooters, P and Q, decide to settle their differences with a paint-ball duel. They will take turns shooting at the other person till one of them is "out", and then the other person is the winner. The probability of a shot of P finding its target is p, and the probability of a shot of Q finding its target is q. Find the probability of P winning when

(a) P shoots first (this is termed P Q duel)

(b) Q shoots first (this is termed Q P duel)

Now we have 3 such paintball shooters, A, B and C. They stand at the vertices of an equilateral triangle, so each person is the same distance from the other two. If you are hit, you are out. If you are the last to not be hit, you win. Each person will try to maximize the chance that they win by not getting hit before the others.

The way they play is that they take turns taking one shot at a time with their paintball gun. A is the worst shot, and when he aims at a target he only hits it 1/3 of the time, so he gets to shoot first. B is second-worst, hitting 2/3 of the time, so he gets to shoot second. C is a sharpshooter who always hits his target, so he will have to wait and shoot third. The three men will take turns shooting their paint-ball guns in this order until only one man is left standing, and he will of course be the winner.

(c) A is up first. What options does he have ? Find his probabilities of survival if he takes those options. What should he do to secure his best odds of survival? (To find this, first find out the best strategy for C and B if all three are "alive" when their turn comes.)

(d) If A chooses the strategy best for him (determined above), what are the chances of survival of B and C ?