

# Math-Circle: Session 1

TIFR-CAM and ICTS

October 24, 2022

## Butterfly effect/Chaotic system

Problem 6. A frog is sitting at the 0 location on the number line. Imagine that the part of the number line after 1 is inside water. The frog wants to reach the water pond by jumping forward. The frog's jumping step size is some number  $a > 0$ . The only problem is that the number line is cursed by the Square root curse: Every time the frog lands on the ground, say at location  $\sqrt{x}$ , it gets immediately transported through an imaginary bridge to the location  $x$ . Show that no matter how small the frog's jumping step size is, it will always reach the pond. Therefore, even if the frog's jumping step size is say  $10^{-100000}$  (which is even smaller than the size of an atom), it will reach the pond. On the other hand, if its jumping step size is zero, which means it does not jump at all, it will of course not reach the pond. Do you see the butterfly effect?

Equivalently, for  $a > 0$ , find the value of

$$\sqrt{a + \sqrt{a + \sqrt{a + \dots}}}$$

What value do you get when you put  $a = 0$ ? What value do you get for  $a > 0$ ? If  $a$  is very very small, are these two values close to each other? What did you learn from this investigation?