Math-Circle: Session 1

October 24, 2022

Think in terms of a Graph

Problem 2   (Friendship Paradox) . Consider a statement:

In any world, where friendship is mutual, the average number of friends people have is at most the average number of friends their friends have!

Just to elaborate, imagine any world with some number of people living in it. In this world, any two people are either friends or not friends, and the friendship is always mutual. The God goes to each person one by one and asks him/her how many friends he/she has. The God then computes the average of collected numbers. The average comes out to be some number L_1. The God again goes to each person one by one and asks him/her what is the average number of friends his/her friends have. The God then again computes the average of collected numbers. This time the average comes out to be some number L_2. Consider the statement

$$L_1 \leq L_2.$$

What do you think about this statement? Is it correct or incorrect? Firstly, try to understand this statement intuitively or by feeling/instinct. What does your intuition/feeling/instinct tell you? Try to convince yourself why you feel the way you do. Can you try to write your thoughts on a piece of paper? Try to construct a mathematical language to express it. Can you try to prove your claim about this statement?