

Math-Circle: Session 1

TIFR-CAM and ICTS

October 24, 2022

Hungry Spider

Problem 4. A spider is very hungry. The spider will get his meal if it is close enough to the fly. Can you see to it that it doesn't go hungry in each of the following situations?

- (a) A spider built a circular web and a fly gets stuck somewhere on it. The spider starts at the centre. At the first step the spider jumps half the radius of the web. After each jump the spider loses its energy and jumps half the distance it could jump earlier. Where can the fly be so that the spider will be able to reach it?
- (b) Suppose this spider does not have a web (Yes! There are many such spiders) and suddenly spots a fly far away. Its steps are of lengths $1, \frac{1}{2}, \frac{1}{3}, \dots$ (You may find it useful to know that even though the step sizes $\frac{1}{2}, \frac{1}{3}, \dots$ are decreasing, their sum becomes arbitrarily large as the number of steps grows).
- (c) The last spider built a square web (I know! This is a very strange spider) and a fly got stuck somewhere on the web. The spider starts anywhere on the web and can jump half the distance to any corner.