

### 3 Abstract Probability

Let  $S$  be a finite set of points in the plane such that no three of them are collinear. For each convex polygon  $P$  whose vertices are in  $S$ , let  $a(P)$  be the number of vertices of  $P$ , and let  $b(P)$  be the number of points of  $S$  which are outside  $P$ . A line segment, a point, and the empty set are considered as convex polygons with 2, 1, 0 vertices respectively. Prove that for every real number  $x$ ,

$$\sum_P x^{a(P)} (1 - x)^{b(P)} = 1,$$

where the sum is taken over all convex polygons with vertices in  $S$ .