

Maths Circle India: Module 8, Session 2
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3 A Palace with Thousand Doors

A palace has 1000 doors all of which were closed at the beginning. There are 1000 security guards in this palace. The first security guard opened all the doors. The second security guard closed the 2nd, 4th, 6th, ..., 1000th doors. The third security guard changed the state of the 3rd, 6th, 9th ..., 999th doors (i.e., he closed the open doors among these and opened the closed ones). The fourth guard changed the state of 4th, 8th, 12th, ..., 1000th doors, and so on. Finally, the thousandth security guard changed the state of the 1000th door.

How many doors were open in the end?