

Maths Circle India: Module 8, Session 4
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1 How Fast Is the Euclidean Algorithm?

Let $1 \leq a < b$ and $f(a, b)$ denote the number of divisions required in the Euclidean Algorithm for computing gcd. Let b_k denote the smallest number such that $f(a, b_k) = k$ for some a .

- Find b_1, b_2, \dots, b_4 .
- Do you get any patterns in (a)? What is this sequence $\{b_1, b_2, b_3, \dots\}$?
- Let $\Phi = (1 + \sqrt{5})/2$ (note that $\Phi^2 = \Phi + 1$). Show that for all a and b ,

$$\Phi^{f(a,b)} \leq b.$$