Maths Circle Explorations: Session 4

TIFR, Mumbai

10th December 2021

Problem 1

Suppose we have 1000 doors in a row marked $1, 2, 3, \ldots, 999, 1000$. All doors are closed. Now, we pick a number n lying between 1 and 1000, and we change the condition of the doors (opening it if it is closed, and closing it if it is open), whose numbers are divisible by n. To elaborate, initially all the doors were closed. We pick 1, and as every number is divisible by 1, we open all the doors. Then we pick 2, and as only even numbers are divisible by 2, we close the even numbered doors, leaving the odd numbered doors open. We continue this process for $3, 4, 5, \ldots, 1000$.

Now the question is, how many doors are open and how many are closed at the end of this process?