

Maths Circle Explorations: Session 4

TIFR, Mumbai

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Problem 2

Imagine yourself to be a farmer in prehistoric times when the concept of numbers hadn't been discovered yet. Suppose you have 10 cows (note: the concept of 10 hadn't been discovered yet) and a fellow farmer has 17 cows. How would you naturally come up with a way to tell that the other farmer has more cows than you have?

By the same line of thought, how would you come up with a way to tell that you have as many eyes as you have ears? (note again : the concept of 2 hadn't been discovered yet)

Can you now come up with an answer to the following questions:

1. Are there as many even numbers as odd numbers?
2. Are there as many natural numbers

$$\mathbb{N} := 1, 2, 3, \dots$$

as integers

$$\mathbb{Z} := \dots, -3, -2, -1, 0, 1, 2, 3, \dots ?$$

3. Are there as many Integers as rational numbers. Rational numbers are written as p/q where p and q are integers and $q \neq 0$?
4. Are there as many straight lines passing through a given fixed point on the plane as there are circles with that fixed point as the centre ?