Maths Circle Explorations: Session 7

TIFR, Mumbai

21st January 2022

Problem 2

1. Show that $\sqrt{2}$ is irrational.
2. Show that $\frac{\sqrt{5}\pm 1}{2}$ is irrational.
3. Is $\sqrt{2}+\sqrt{3}+\sqrt{5}$? irrational ?
4. If $p+q+r$ is rational and $\sqrt{p}+\sqrt{q}+\sqrt{r}$ is also rational. Can p or $\sqrt{p}$ be irrational ?

How do you calculate square root ? Can you write your method and prove its correctness ?

Consider the stated method to find the square root of N.

Make an initial guess $x(0)$ of the square root of N. (i.e. $x(0)^2 \sim N$)

Next assume that you have already found $x(1), x(2), \ldots, x(n)$. Now you find $x(n+1)$ by the operations below.

$$\begin{aligned} a(n) &= \frac{N - x(n)^2}{2x(n)} \\ b(n) &= x(n) + a(n) \\ x(n+1) &= b(n) - \frac{a(n)^2}{2b(n)} \end{aligned}$$

Assertion: The numbers $x(n)$ as n increases come closer and closer to the square root of the number N.

Prove or disprove the Assertion.