Math Circle Explorations: Session 2
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Problem 1. Consider a stick of a fixed length L.

(i) A person breaks the stick into some pieces. How do you arrange those pieces so that area enclosed by the arrangement is maximum?

(ii) If you are given the option of breaking the stick into ' n^1 number of pieces how would you do that so that the area enclosed by them is largest among all possible ways.

(iii) Once you have found the optimal way of breaking the stick into a fixed number of pieces to maximize the area. We can keep increasing the number of pieces and arrange them to get the largest area. How can you compare the largest area obtained with the length of pieces of the stick? Will this process ever end?

(iv) Suppose you are given a rope oflength L. How will you join the two ends of the rope so that the area enclosed in the loop is largest? Can we always find a solution to this problem?

(v) What is the largest volume enclosed by a bubble of surface area A?

(vi) Draw a region in a plane such that a unit needle can be placed in it. Can you rotate the needle continuously full 180 degrees within the same region such that it returns to its original position with ends reversed ? Give examples of regions where it is possible to do it.