Math Circle Explorations: Session 2
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Problem 2.   The students in a classroom have formed numerous clubs. Every club needs to have a president. A student can be a member of multiple clubs. The teacher wants to appoint a president for each club in such a way that no student is the president of more than one club. Clearly, it is not always possible to pick a president for each club while satisfying this requirement. For example, if there are 60 students in class but 61 clubs in all, we cannot pick distinct presidents for each club. So, whether this problem is solvable or not depends on the number of these clubs and also their membership.

   How can the teacher figure out whether the problem (of picking distinct presidents for each club) is solvable or not? Ifit is solvable, can we formulate a systematic method (an "algorithm") by which this problem can be solved?