DTP-Math-Circle: Session 4—Probability, Inequalities and Quantum Mechanics

Oct 14 2022

6 Leggett-Garg inequality

You find a gadget which has three displays and a button. When you press the button, you see that the first display lights up and shows a number, either +1 or −1, then after a little while, the second lights up and shows another number, again either +1 or −1, and finally after another pause, the third display does the same thing. And after all three numbers are displayed, the gadget announces that it has wiped its memory clean and reset itself to its original state. You are now free to press the button again. Each time you press the button, a fresh sequence of three numbers seems to appear in the three displays.

Let us call the three numbers that appear in the three displays A, B, and C respectively. You repeat this "experiment" of pressing the button a large number of times, say N. And you measure

$$E_{AB} = \frac{1}{N}\sum_{i=1}^{N} A_i B_i$$

where A_i and B_i are the numbers that appear in the first and second display in the i^{th} run of this "experiment". And we measure E_{BC} and E_{AC} similarly.

Having measured these quantities, you look at the combination

$$K = E_{AB} + E_{BC} - E_{AC}$$

Since there doesn't seem to be a fixed outcome, you start thinking in terms of a probability $P(A, B, C)$ for getting each possible sequence of numbers A, B, C. In terms of $P(A, B, C)$, what is your prediction for E_{AB}, E_{BC}, and E_{AC} when the number of measurements N you make is very large?

Using this, can you place bounds on the range of K?